Economics 203: Section 7

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1 Logistics

Don't be shy about coming to office hours! They are Thursday, 4-6 pm in Landau 360.

2 Concepts

We have no new solution concepts this week, so I will just briefly summarize the applications that were discussed in the last two weeks of lecture:

- Competition
- Sequential bargaining

2.1 Competition

Most of section 5 of Doug's notes focused on issues of competition, including leader-follower, price competition with capacity constraints, and entry deterrence. It is hard to summarize all these settings, except to note that the standard Bertrand and Cournot results can be very sensitive to the addition of new dimensions that firms can compete on. I would not try to memorize all these results, but be comfortable with the general solution concepts and problem solving techniques.

2.2 Sequential Bargaining

2.2.1 In Theory

We imagine that two player are splitting a pie of size 1. They take turns making offers, which the other player can then accept or reject. If the split $(x, 1-x)$ is implemented in period t, then the payoffs are $(\delta_1^t x, \delta_2^t(1-x))$. The game continues indefinitely until agreement is reached, with the players alternating who makes the offer and who chooses to accept or reject.

In Doug's notes, it is shown that the game ends immediately, with player i (the player making the first offer) getting share

$$\frac{1-\delta_i}{1-\delta_i\delta_j}.$$

I will not reproduce the proof here, but I will point out that the proof only checks one-period deviations from the SPNE. That is, we only need to check that changing an offer at the beginning of a subgame is not a profitable deviation for that player, and that changing the ensuing accept-reject decision cannot be profitable for the other player.

We can do this because of the repeated nature of the game. Suppose we didn't find a profitable one-time deviation for player i, but we did find it is a profitable deviation for player i to change her offer in the subgame at round t, and then to *also* change her accept/reject threshold in the following round, $t+1$. Then one of two things is true:

- The game still ends in after round t. Then that one-time deviation in round t must have been profitable *without* the ensuing deviation in round $t+1$. Thus, we have a contradiction.
- The game does not end after round t. Player j makes the SPNE offer, regardless of player i deviating in the previous round. Thus if player i now deviating from the SPNE is profitable, it must have also been profitable as a standalone deviation. Thus, we have a contradiction in this case as well.

By this logic, we see that ruling out any profitable one-time deviations will rule out more complicated deviations, as long as the SPNE strategies are stationary.

2.2.2 In Practice

Note that in both the finite and infinite horizon versions of this game, the SPNE outcome is such that the split is agreed upon without delay. This is not, however, what happens in applied settings or in the lab. In fact, we often see a strong deadline effect, where players do not reach agreement until the last few seconds of the game. We even see that players sometimes do not reach an agreement at all.

3 Problems

Problem 1. *Problem bank #68.*

Suppose that there are I consumers, each with demand given by $q_i(p) = a - bp$ $(a, b > 0)$. Firms incur a cost K when they enter the industry, and a variable cost c for each unit produced. There are infinitely many (identical) potential firms.

(a) Suppose that all firms make entry and production choices (quantities) simultaneously in a single stage. What happens to equilibrium prices, output levels, and consumer surplus as I goes to infinity?

(b) How do your answers in part (a) change if firms first make entry decisions simultaneously, and then, having observed each others' entry decisions, choose quantities (assuming that the entry cost, K, is sunk)?

Problem 2. *Problem bank #72.*

Consider the bargaining model discussed in class, but instead of assuming that players discount future payoffs, assume that it costs $c < 1$ to make each offer. (Only the player making the offer incurs this cost, and players who have made offers incur this cost even if no agreement is ultimately reached.) Assume first the horizon is finite, of length T.

(a) What is the (unique) SPNE of this alternative model?

(b) What happens as T approaches infinity?

(c) Now suppose that the horizon is infinite. What are the subgame perfect Nash equilibria of this game?

4 Solutions

Solution 1.

(a) We can see that we will have three conditions for NE: that the firms entering to not want to exit, that the firms entering are choosing the optimal quantity, and than the non-entering firms to not wish to enter. We consider these three conditions in turn.

- Optimal quantity: Note that total demand in the economy is $Q = I(a - bp)$, or equivalently $p = \frac{a}{b} - \frac{Q}{bI}$. Thus a single firm's optimal quantity is given by

$$q_i = \arg\max_q \left(\frac{a}{b} - \frac{q + q_{-i}}{bI} - c \right) q,$$

for which the FOC gives $\frac{a}{b} - c - 2\frac{q_i}{bI} - \frac{q_{-i}}{bI} = 0$. Noting that $q_{-i} = (N-1)q_i$ by symmetry, we find that $q_N = (a-bc)\frac{I}{N+1}$, where we include the N subscript to emphasize that this is the quantity when there are N firms in the market.

- Optimal entry: From the quantity above, we can calculate the market price: $p_N = \frac{a}{b} - \frac{1}{bI}\frac{N(a-bc)I}{N+1} = \frac{a}{b} - \frac{1}{b}\frac{N(a-bc)}{N+1}$. From here we can calculate the profit for the entering firms: $\pi_N = \left[\frac{a}{b} - \frac{1}{b}\frac{N(a-bc)}{N+1} - c\right](a - bc)\frac{I}{N+1} = \frac{(a-bc)^2 I}{(N+1)^2 b}$. For entering firms to not wish to deviate out of the market, we must have $\frac{(a-bc)^2 I}{(N+1)^2 b} - K \geq 0$, since they pay cost K for entering.
- Optimal non-entry: Note that if a non-entrant were to deviate to entering, that firm would face N other firms, all producing quantity q_N. Thus the deviating firm's optimal quantity would solve

$$q_D = \arg\max_q \left(\frac{a}{b} - \frac{q + Nq_N^C}{bI} - c\right) q,$$

which we can solve to find that $q_D = \frac{(a-bc)I}{2}\frac{1}{N+1}$. Then the total quantity is $Q = Nq_N + q_D = N(a-bc)\frac{I}{N+1} + \frac{(a-bc)I}{2}\frac{1}{N+1} = \frac{2N+1}{2}\frac{a-bc}{N+1}I$, and the new price is $p = \frac{a}{b} - \left(\frac{a}{b} - c\right)\frac{2N+1}{2(N+1)}$. This finally allows us to calculate the profit for the deviating firm: $\pi_D = \frac{(ab-c)^2}{4b}\frac{1}{(N+1)^2}I$. The non-entering firm will thus prefer to stay out if $\frac{(ab-c)^2}{4(N+1)^2 b}I - K \leq 0$.

Thus we have show that N firms entering and producing $q_N = \frac{a-bc}{N+1}I$ is an equilibrium if

$$\frac{1}{4}\frac{(ab-c)^2}{(N+1)^2}\frac{I}{b} \leq K \leq \frac{(ab-c)^2}{(N+1)^2}\frac{I}{b}$$

We are now asked to find the limits of some important values as $I \to \infty$.

- Note that for fixed K, the entry and non-entry conditions restrict the ratio of N and I to be bounded above and below. Thus as $I \to \infty$, $N \to \infty$ as well. Then $p_N = \frac{a}{b} - \frac{1}{b}\frac{N(a-bc)}{N+1} \to c$.
- Total output for is $Nq_N = (a-bc)\frac{IN}{N+1} \to \infty$.
- Total surplus is $CS = \frac{1}{2}\left(\frac{N}{N+1}\right)^2\frac{(a-bc)^2}{b}I$. You can calculate this easily by noting that since demand is linear, surplus is just a triangle. Surplus goes to infinity, but note that surplus per consumer has a finite limit: $\frac{CS}{I} = \frac{1}{2}\left(\frac{N}{N+1}\right)^2\frac{(a-bc)^2}{b} \to \frac{1}{2}\frac{(a-bc)^2}{b}$.

(b) Note that we still have three conditions for a SPNE: that entering firms pick optimal prices, that entering firms do not wish to exit, and that non-entering firms do not wish to enter. The first two conditions are the same as above, but the third condition changes, since now a firm deviating from non-entry to entry will be observed by its competitors before they all name prices. Thus this firm should pick the $N+1$ Cournot quantity, not the quantity we calculated above. Its profit upon entry will just be the $N+1$ Cournot profit. Thus the non-entry condition becomes $\frac{(ab-c)^2}{(N+2)^2 b} I - K \leq 0$.

Note that the equilbrium path has not changed, so the limits calculate in the previous part are the same here.

Solution 2.

(a) Since the game is finite, we can solve by backwards induction. Throughout, let the x be the amount that player 1 gets in any split, so that the payoffs are $(x, 1-x)$ (not including any costs incurred). First, we assume that T is even.

Period T: Player 2 makes the offer in the final period. Player 1 will accept any offer, so 2 offers $x = 0$, which is accepted. Thus the payoffs *in this round only* are $(0, 1-c)$, since 2 incurs cost c for making an offer.[1]

Period T-1: Player 1 makes the offer, and 2 accepts or rejects. Note that 2's continuation payoff for rejecting is $1-c$, so he will accept iff $1-x \geq 1-c$. 1's continuation payoffs in this case will be 0, so his best response is to offer $1-x = 1-c$ to 2, which is accepted. Note that payoffs in this round are $(0, 1-c)$, since 1 gets c of the pie buy pays c for making the offer, netting him 0.

Period T-2: Player 2 makes the offer, and 1 accepts or rejects. Not that 1's continuation payoff for rejecting is 0, so he will accept any offer. Thus 2 proposes $x = 0$, which accepted. Yet again, payoffs in this round are $(0, 1-c)$.

We can now see that continuation payoffs will *always* be $(0, 1-c)$, so the steps above will repeat until the reach the beginning of the game. Thus we have that the following strategy profile is the unique SPNE of the game (when T even):

- Player 1 proposes $(c, 1-c)$ in every period where he gets to make an offer.
- Player 1 accepts an offer iff $x \geq 0$.
- Player 2 proposes $(0, 1)$ in every period where he gets to make an offer.

[1] All the costs up until this point are sunk, so we don't need to consider them when figuring out which actions the players will take in this subgame.

- Player 2 accepts an offer iff $1 - x \geq 1 - c$.

In this case, the game ends immediately, and payoffs are $(0, 1 - c)$.

Next, we consider the case of T odd. But note that by the same backwards induction argurment, we get that the roles of the players are just switched. Thus we have that the following strategy profile is the unique SPNE of the game (when T odd):

- Player 2 proposes $(1 - c, c)$ in every period where he gets to make an offer.
- Player 2 accepts an offer iff $1 - x \geq 0$.
- Player 1 proposes $(1, 0)$ in every period where he gets to make an offer.
- Player 1 accepts an offer iff $x \geq 1 - c$.

Again in this case, the game ends immediately, and payoffs are $(1 - c, 0)$.

(b) The outcome depends only on the parity of T, and so he payoffs do not converge as T goes to infinity. Compare this to the discounting case, where the payoffs do converge as T goes to infinity.

(c) Note that the solution to the finite horizon version was *stationary*; that is, the specified strategies did not depend on the period. Thus, we can guess that there is a stationary strategy for the infinite horizon version as well, which also ends the game immediately. Such an equilibrium would take the following form:

- Player 1 proposes $(a, 1 - a)$ in every period where he gets to make an offer.
- Player 1 accepts an offer iff $x \geq b$.
- Player 2 proposes $(b, 1 - b)$ in every period where he gets to make an offer.
- Player 2 accepts an offer iff $1 - x \geq 1 - a$.

For this proposed profile to be an equilibrium, we need player 2's accept and reject decisions to be optimal. In particular, we need player 1 to be better off accepting 1's offer of $1 - a$ than rejecting it. Thus we must have $1 - a \geq 1 - b - c$, or $b \geq a - c$. We also need the payoff to 2 for rejecting an offer $1 - a - \epsilon$ for any $\epsilon > 0$ to be optimal. That is, we need $1 - a - \epsilon \leq 1 - b - c$ for all $\epsilon > 0$. Thus we must have $b \leq a - c$. Combining these results we find that $b = a - c$. Thus I propose that the following strategy profile is a SPNE, for any $a \in [c, 1]$:

- Player 1 proposes $(a, 1 - a)$ in every period where he gets to make an offer.
- Player 1 accepts an offer iff $x \geq a - c$.

- Player 2 proposes $(a-c, 1-a+c)$ in every period where he gets to make an offer.
- Player 2 accepts an offer iff $1-x \geq 1-a$.

To check that this is in fact a SPNE, it suffices to check one-shot deviations by each player.

- In equilibrium, 1 offers $(a, 1-a)$, which is accepted, giving him a payoff of $a-c$. If 1 were to increase his offer to 2, 2 would accept and 1 would get a lower payoff. If 1 were to decrease his offer, 2 would reject, and in the ensuing period 1 would accept 2's offer of $a-c$. 1's total payoffs would be $a-c-c < a-c$, so this deviation is not profitable. Thus 1 will not deviate from his equilibrium offer.
- In equilibrium, 2 offers $(a-c, 1-a+c)$, which is accepted, giving him a payoff of $1-a+c-c = 1-a$. If 2 were to increase his offer to 1, 1 would accept and 2 would get a lower payoff. If 2 were to decrease his offer, 1 would reject, and in the ensuing period 2 would accept 1's offer of $1-a$. 2's total payoffs would be $1-a-c < 1-a$, so this deviation is not profitable. Thus 2 will not deviate from his equilibrium offer.
- Consider 1's accept/reject decision. If he rejects, he gets payoff $a-c$ in the ensuing round. If he accepts some offer x he gets payoff x. Clearly he should accept iff $x \geq a-c$, as specified in the equilibrium.
- Consider 2's accept/reject decision. If he rejects, he gets payoff $1-a+c-c = 1-a$ in the ensuing round. If he accepts some offer $1-x$ he gets payoff $1-x$. Clearly he should accept iff $1-x \geq 1-a$, as specified in the equilibrium.

No player has incentive to deviate, so the strategies above constitute a SPNE.