Economics 203: Section 5

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1 Logistics

1.1 Mid-Quarter Section Review

You should have gotten an email from me with a survey about this section. Please fill this out before midnight tonight! It should only take a few minutes, but it will help me immensely in making section better for you.

1.2 Problem Sets 2 and 3 Graded

Problem Set 2 and 3 have been graded and are available for pick up in section and in lecture.

Some comments on individual problems:

- Problem 5: Note that if you have a loop of best-response relations, then every pure strategy in that loop is rationalizable, no just the strategy you happened to start with. Also, remember that strict dominance implies weak dominance.
- Problem 6: Many people missed the equilibrium where both players make a demand greater than or equal to \$100. (However, this may be due to some of you assuming the strategy space was bounded above by \$100; this was unclear in the problem, so I've fixed that ambiguity for future years.)
- Problem 9: This problem highlights how much trouble you can save in proofs by using symmetry and generality to your advantage. For example, you can assume without loss of generality that $x_1 \leq x_2 \leq x_3$. By symmetry, if you prove that there are no NE when $x_3 \leq \frac{1}{2}$, say, then you have also proved that there are no NE when $x_1 \geq \frac{1}{2}$.
- Problem 11: Read carefully and answer fully! Many people didn't state the NE quantity produced or give its limits.

- Problem 16: Remember, a PSNE is a MSNE. Also, when looking for MSNE, don't forget to check all possible combinations of mixing (1 by 2, 2 by 2, 1 by 3, etc).
- Problem 19: Don't forget to note information sets when drawing the extensive form of a game.

General comments:

- You can really help me by summarizing your answers and boxing or underlining key results.
- This is not a contest to see who can fit their problem set on a single page.
- Attach your problems and pages in order, please.
- Make sure you are doing the right problems. Some of you did problems that were not assigned, which may be due to working of an outdated problem bank. Also, check with your classmates!
- Try to complete all the parts of all problems. If you get stuck, come to office hours, email me, or work with your study group.

2 Concepts

2.1 Subgame Perfect Nash Equilibrium

By this point in the class we have seen that Nash equilibrium sometimes makes predictions that are not "sensible." In extensive games, this is made especially evident by the fact that Nash equilibrium allows players to make non-credible threats, as in the entry game discussed in lecture. Such self-defeating punishments get by because Nash equilibrium only restricts players to play sensibly on the equilibrium path. If we instead force players to make sensible choices at more points in the game, then we have a more restrictive solution concept that will (hopefully) make more reasonable predictions.

Definition 1. Let t be a node in a dynamic game, $h(t)$ be the info set that contains t, and $S(t)$ be all the successor nodes of t. Then a **proper subgame** of a game is a set of nodes $t \cup S(t)$ along with all mappings from info sets to players, from branches to actions, and from terminal nodes to payoffs, such that

(i) $h(t) = t$ and

(ii) for all $t' \in S(t)$, $h(t') \subseteq S(t)$.

That is, t is a singleton information set, and $S(t)$ (all the nodes following t) contains all the info sets it intersects.

Visually, we can imagining "pruning" the game tree just above the node t. If we can remove this pruned branch without "tearing" any information sets, then we have a valid subgame.

Definition 2. A Nash equilibrium δ^* in behavior strategies is **subgame perfect** iff for every proper subgame, the restriction of δ^* to that subgame is a Nash equilibrium in behavior strategies.

2.1.1 Finding Subgame Perect Nash Equilibria

First, let's consider finite[1] games of perfect information, where every node is an information set, and so every node is the start of a subgame. Further, let's assume the game is *generic*, meaning that no player is indifferent between any of her payoffs at the terminal nodes. Thus we can use backwards induction to find every SPNE:

- Start at the last non-terminal nodes (i.e. the last nodes where a decision is made). For the player making the choice at each of these nodes, note that if they were to find themselves at this node, they would simply select the highest-payoff option from their available actions.
- Move up the tree one branch. The players acting at these nodes choose the best option by looking at the so-called "continuation" payoffs; that is, they select their highest-payoff option, knowing what their opponents will do after each of their actions by the previous step.
- Continue this process all the way up the game tree.

This should make it clear that a generic, finite game of perfect information has a unique SPNE.

Note, however, that we can use this same approach even if the game in not generic. The only wrinkle is that there may be multiple SPNE, since players may be indifferent between continuation payoffs.

We can even use backwards induction on finite games of *im*perfect information. In these cases, you'll need to imagine subgames that involve non-singleton information sets as games in normal form. Find the (possibly not unique) NE of these games to find the (possibly not unique) continuation payoffs.

Lastly, note that we can use backwards induction for games that have an infinite number of histories but the maximum length of a history is bounded (that is, games of *finite horizon*). In these games, you'll need to be very careful about properly labeling and describing strategies. Suppose, for example, that player 1 moves first in a game with an action $x \in \mathbb{R}$; then there are an infinite number of subgames, one for each possible x. Player 2's strategy needs to describe his actions in every subgame. That is, his strategy will depend on x in general. A

[1] Remember that by finite, we mean that there is a finite number of possible histories in the game.

strategy profile is then a subgame perfect Nash equilbrium in this game if the specified strategy for player 2 is optimal after *every* x, even if this particular x is never played in equilibrium by player 1.

2.2 Looking Ahead

Note that NE essentially only restricts that "sensible" choices (that is, best responses) be made at the beginning of a game, when players select their strategies. SPNE further restricts that sensible choices be made at any singleton information sets, but leaves open the possibility of strange decisions being made when players have imperfect information. Perfect Bayesian equilibrium restricts that sensible choices be made at every information set, conditional on beliefs; but these beliefs can be themselves non-sensical if they are off the equilibrium path. Finally, sequential equilibrium restricts that these rationalizing beliefs be consistent.

3 Problems

Problem 1. *The ultimatum game.*

Consider the following situation, wherein two players split a pie. Player 1 proposes a split, and then player 2 accepts or rejects this split. If player 2 rejects the split, both players get nothing.

(a) Model this situation as a game in extensive form.

(b) What payoffs can be supported by Nash equilibrium?

(c) Find the subgame perfect Nash equilibria. What payoffs can be supported in this case?

Problem 2. *Five pirates.*

Five pirates have discovered 100 gold coins worth of treasure. They agree to split the treasure by Blackbeard's Rules: The highest-ranked pirate proposes an allocation. All pirates then vote on this allocation simultaneously. If a weak majority approve, the allocation is made; otherwise, the proposer walks the plank and the process is repeated with the next-highest-ranked pirate. Note that pirates enjoy making others walk the plank, though not as much as they like a single gold coin. Give a SPNE outcome of this game.[2]

Problem 3. *Problem Bank #46.*

[2]Previously this problem asked for *the* SPNE outcome. As we saw in class, there is more than one possible PSNE outcome, so I have updated the wording of the question.

Nation A plans to attack nation B. The attack can occur at one of two locations, C and D. The success or failure of the attack depends on three factors: where A's troops are massed prior to the attack (near C or near D), where the attack occurs, and which location is defended. Let x denote the fraction of A's troops massed near C, and let $1-x$ denote the fraction of A's troops massed near D. The game is played as follows: simultaneously, A chooses to attack either C or D, and B decides to defend either C or D. Payoffs are determined as follows. If the attack is successful, the payoff to A is 1, and the payoff to B is zero. If the attack is unsuccessful, the payoff to A is zero, and the payoff to B is 1. The probability of a successful attack is in turn determined as follows. Let z denote the fraction of A's troops massed near the location where A attacks (so $z = x$ if A attacks at C, and $z = 1 - x$ if A attacks at D). Then the probability of a successful attack is z if B does not defend the location where A attacks, and $\frac{z}{2}$ if B does defend the location where A attacks.

(a) Depict the normal form of this game. Are there ranges of x for which either or both players have dominant strategies? If so, indicate which strategies are dominant over which ranges. Are there ranges of x for which pure strategy Nash equilibria exist? If so, indicate the ranges, and characterize the Nash equilibria (specify equilibrium strategies and indicate payoffs). Are there ranges of x for which mixed strategy Nash equilbria exist? If so, indicate the ranges and characterize the mixed strategy Nash equilibria (specify equilibrium strategies and indicate payoffs). Draw a graph representing A's expected equilibrium payoff as a function of x.

(b) Now imagine that, instead of being fixed, x (the deployment of troops) is under A's control. Events occur in the following order: first, A decides how many troops to mass near each location (that is, A chooses x); next, B observes the deployment of A's troops (that is, B observes x); finally, A chooses to attack either C or D, and simultaneously B decides to defend either C or D. Solve for the subgame perfect equilibria of this game. How does A deploy its troops?

Problem 4. *Problem bank #48.*

Players 1 and 2 each begin with an endowment of one dollar. Player i decides whether to contribute this dollar to a public good, for $i = 1, 2$. The contribution yields benefit a for each player. If both players contribute, both receive benefit $2a$. Contribution decisions are made simultaneously. Assume that $0 < a < 1 < 2a$.

(a) Assume that players care only about their own payoffs. Identify all pure strategy Nash equilibria.

(b) Now suppose that after the contribution stage, actions are observed and each player has a chance to punish his opponent. By paying $p \geq 0$ dollars, he reduces his opponent's monetary payoff by Kp where $K > 1$.

Punishment choices are made simultaneously. p is not restricted by stage 1 payoffs. Any $p \geq 0$ is allowed. Solve for the pure strategy subgame perfect Nash equilibria of this game.

(c) Now suppose that instead of caring solely about monetary outcomes, the payoff of player i is given by

$$g_i(x_i, x_j) = x_i - \alpha_i \max[x_j - x_i, 0] - \beta_i \max[x_i - x_j, 0]$$

where $\alpha_i \geq \beta_i, \beta_i \in [0, 1)$ for $i = 1, 2$ and x_i and x_j are the monetary outcomes for i and j. α_i and β_i are common knowledge.

(i) Interpret these payoffs.

(ii) Repeat part (a) using these payoffs. How do your answers depend on the parameters? Interpret.

(iii) Repeat part (b) using these payoffs. When considering the punishment subgames, restrict attention to equilibria in which at most one player punishes. Interpret the equilibria.

4 Solutions

Solution 1.

(a) Let the pie be of size 1. Then player 1's strategy is to choose $x \in [0, 1]$. Player 2's strategy is a function $f : [0, 1] \to \{Accept, Reject\}$. Note that player 2's strategy gives his accept/reject decision for every possible split of the pie. If 2 accepts, payoffs are $(x, 1 - x)$. If he rejects, payoffs are $(0, 0)$.

(b) Consider the following strategy profile: Player 1 picks $x = k$ and player 2 accepts iff $x = k$, for some $k \in [0, 1]$. Note that player 1 is getting $k \geq 0$, whereas any deviation will get him payoff 0, so he has no incentive to deviate. Player 2 also has no incentive to deviate, since no strategy can get him more than $1 - k$, given that 1 is playing $x = k$. Thus we have a NE. So, we see that any payoffs of the form $(k, 1 - k)$ for $k \in [0, 1]$ can be supported by a NE.

Note also that the strategy profile where 1 plays $x = 1$ and 2 always rejects is also a NE. Both players get 0 in equilibrium, and can do no better by deviating. Thus the payoffs $(0, 0)$ can also be supported by a NE.

(c) Consider a subgame where 1 has just played $x = k$. If $k > 0$, 2's unique best response is to accept. Thus any SPNE strategy profile must have $f(k) = Accept$ for $k > 0$. If $k = 0$, then 2 is indifferent between accepting and rejecting, so either is a best response.

Now consider player 1's choice at the beginning of the game. Any $x = k > 0$ will get him payoff $k > 0$. Thus the strategy profile where $x = 1$ and 2 always accepts is a SPNE. The only other possible SPNE is one where 2 rejects iff $x = 0$, but in this case 1's best response does not exists, so we can't have a SPNE. Thus in the only SPNE of this game, the payoffs are $(1, 0)$.

Solution 2.

Let's call the 5 pirates A, B, C, D, and E, ranked in that order. The key to this game is to note that while there are millions of subgames, they can be grouped according to who is making the decision at the beginning of that subgame. At any subgame where a particular pirate is making his allocation decision, all the pirates ranked above him are dead and all the coins remain to be allocated. Whatever proposals were made before have no effect on the subgame.

First, consider the subgame where pirate E proposes an allocation. He clearly allocates everything to himself, since he is the only surviving pirate.

Next, consider the subgame where pirate D proposes an allocation. He knows that he can win any vote by a tie, so he allocates everything to himself and votes for this allocation. Pirate E's vote has no effect on the outcome in this case, so he can vote up or down. The payoffs are 100 for D and 0 for E.

Next, consider the subgame where pirate C proposes an allocation. Note that if his proposal is not passed, he will walk the plank, and the continuation payoffs will be 100 for D and 0 for E. Note that by allocating at least 1 coin to E, C can guarantee that his proposal will pass, since he and E will vote for it.[3] Thus in this subgame the payoffs are 99 for C, 0 for D, and 1 for E.

Next, the subgame where pirate B proposes. By the same logic as above, he notes that he must get at least one other pirate to vote up his proposal, and the cheapest way to do this is to give 1 coin to pirate D, who would otherwise get 0 in continuation. Thus the payoffs are 99 for B, 0 for C, 1 for D, and 0 for E.

Finally, we arrive at the top of the game tree. Note that 1 needs the support of two other pirates to avoid walking the plank. But noting that C and E get 0 in continuation if A's proposal is rejected, A can offer just 1 coin each to C and E to get their support. Thus a SPNE outcome of this game is that A gets 98 coins, B gets 0, C gets 1, D gets 0, and E gets 1.

Note. Throughout this solution I have been a bit loose in describing the full voting strategies of the pirates. I have assumed that the pirates always vote for their preferred continuation payoff. Note that this is not the only possibility for SPNE. For example, in the first voting round, it is possible to have all pirates

[3] Here the tiebreaking rule is important: If he were to get 0 coins either way, E would rather make D walk the plank.

vote for any allocation that pirate A offers. This is a NE of this voting subgame, since no pirate can affect the outcome by changing their vote. Given this, pirate A can choose to take all the gold for himself. So, this is another possible PSNE outcome.

Solution 3.

(a) First, we need to construct the normal form, which is given in Figure 1. Note that since payoffs are 1 if a nation is successful and 0 if not, then payoffs for a particular strategy profile are just the probability of success for each nation. Note also that since x is fixed for this part, and nations attack/defend simultaneously, each nation has just 2 strategies: attack/defend C and attack/defend D.

	C	D
C	$(\frac{x}{2}, 1-\frac{x}{2})$	$(x, 1-x)$
D	$(1-x, x)$	$(\frac{1-x}{2}, \frac{1+x}{2})$

Figure 1: The normal form of the game in part (a). Nation A chooses rows and Nation B chooses columns.

The answers to this part are summarized in Table 1.

x	Dominant strat.	PSNE	MSNE	NE payoffs
$< 1/3$	D for A	(D, D)	none	$\left(\frac{1-x}{2}, \frac{1+x}{2}\right)$
$= 1/3$	none	(D, D)	$\left\{((p, 1-p), D) : p \leq \frac{2}{3}\right\}$	$(1/3, 2/3)$
$\in (1/3, 2/3)$	none	none	$((1-x, x), (3x-1, 2-3x))$	$\left(\frac{3x(x-1)}{2}, 1-\frac{3x(x-1)}{2}\right)$
$= 2/3$	none	(C, C)	$\left\{((p, 1-p), C) : p \geq \frac{1}{3}\right\}$	$(1/3, 2/3)$
$> 2/3$	C for A	(C, C)	none	$\left(\frac{x}{2}, 1-\frac{x}{2}\right)$

Table 1: Summary for part (a).

To see the dominant strategies results, note that for C to be dominant for player A, we must have $\frac{x}{2} > 1-x$ and $x > \frac{1-x}{2}$. These conditions are jointly satisfied iff $x > \frac{2}{3}$. By a similar method we can show that D is dominant for A iff $x < \frac{1}{3}$. For C to be dominant for player B, we need $1-\frac{x}{2} > 1-x$ and $x > \frac{1+x}{2}$. But these conditions cannot both be true for $x \in [0, 1]$, so C can never be dominant for B. Similarly, we can show that D can never be dominant for B as well.

Next we look for PSNE. Note that if $x < 1/3$ the game is dominance solvable and the unique NE is (D, D). Similarly, the unique NE for $x > 2/3$ is (C, C). Payoffs are $(\frac{1-x}{2}, \frac{1+x}{2})$ and $(\frac{x}{2}, 1-\frac{x}{2})$, respectively. One can also verify directly that for $x = 1/3$, (D, D) is a PSNE (with payoffs $(1/3, 2/3)$), and for $x = 2/3$, (C, C) is a PSNE (with payoffs $(1/3, 2/3)$). Lastly, for the range $x \in (1/3, 2/3)$, one can show that there are no PSNE, since $\frac{x}{2} > 1-x$, $x < \frac{1+x}{2}$, $1 - \frac{x}{2} > 1-x$, and $x < \frac{1+x}{2}$ in that range.

Finally, we need to look for MSNE where applicable. Consider the case $x = 1/3$. One can show easily that a 2-by-2 MSNE is not possible. The only possibility for a 2-by-1 MSNE is that player B plays C and A mixes with weight p on C. A is clearly best-responding, since he get payoff $1/3$ from C and from D. Then for B to be best-responding, we must have $p \geq 1/3$. Thus we have the family of MSNE $\left\{((p, 1-p), D) : p \leq \frac{2}{3}\right\}$, where payoffs are $(1/3, 2/3)$ for all these MSNE. Similarly, we can show that for $x = 2/3$, we have the family of MSNE $\left\{((p, 1-p), C) : p \geq \frac{1}{3}\right\}$, where again payoffs are $(1/3, 2/3)$ for all these MSNE. And lastly, we consider the case $x \in (1/3, 2/3)$. Note that 2-by-1 MSNE are not possible, since all best-responses are strict. Thus, direct calculation by the usual methods gets the MSNE $((1-x, x), (3x-1, 2-3x))$, with payoffs $\left(\frac{3x(1-x)}{2}, 1 - \frac{3x(1-x)}{2}\right)$.

The graph of A's payoffs as a function of x is given in Figure 2.

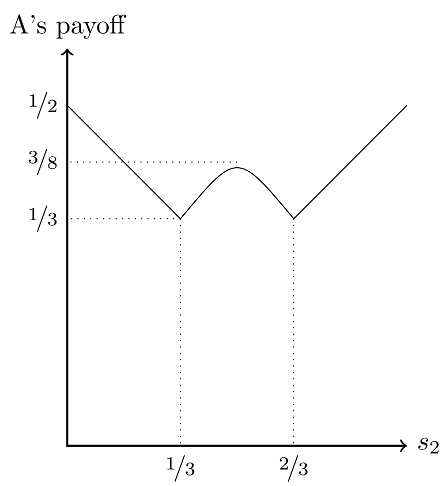

Figure 2: Nation A's NE payoff as a function of x.

(b) Note that the game has one proper subgame for each possble x, but we've solved for the NE in all of these games in the previous part. Thus nation A simply chooses the continuation game to play (i.e. the x) that gives it the highest NE payoff in the corresponding subgame. As our graph in the previous part makes clear, nation A can choose either $x = 1$ followed by C or $x = 0$ followed by D for a payoff of $\frac{1}{2}$ in either case. That is, A masses all of its troops at one location, and then attacks that location.

Solution 4.

(a) The normal form of the game is as follows:

	c	n
c	$2a, 2a$	$a, 1+a$
n	$1+a, a$	$1, 1$

Note that not contributing (ie playing n) is a dominant strategy for both players since $a < 1$, so the unique PSNE is (n, n).

(b) Punishing can only reduce one's own payoffs, so in any subgame following the initial stage, no punishment will occur in equilibrium. Thus the unique SPNE strategy for both players is to play n and always choose $p = 0$.

(c) (i) These new utility functions show that the players care about equity of money. In particular, they are hurt if either player has more money that the other, and the pain is greater if they are the one who has less money.

(ii) The normal form is as follows:

	c	n
c	$2a, 2a$	$a - \alpha_1, 1 + a - \beta_2$
n	$1 + a - \beta_1, a - \alpha_2$	$1, 1$

We can see that (n, n) is an equilibrium, since $1 > a - \alpha_i$ for $i \in 1, 2$. For this same reason, (n, c) and (c, n) can't be equilibria. What about (c, c)? For both players not to have an incentive to deviate, we need $2a > 1 + a - \beta_i$, or $\beta_i > 1 - a$, for $i \in 1, 2$.

Thus in summary the NE are

- (c, c) iff $\beta_i > 1 - a$, for $i \in 1, 2$, and
- (n, n).

(iii) We will first find the NE of every subgame, and then use this to find the SPNE of the whole game. Note that the set of possible subgames can be parameterized by the wealth levels (Y_1, Y_2) of the two players after the contribution stage. As mentioned in the directions, we will restrict attention to the cases where at most one player chooses to punish.

First consider the case $Y_1 = Y_2$. Note that no punishment is optimal, since punishment would lower one's wealth *and* increase inequality, which decreases one's payoff.

Next consider the case $Y_1 > Y_2$. We have two sub-cases to check:

- Is $p_1 > 0, p_2 = 0$ a NE of this subgame? In this case, 1's utility is

$$Y_1 - p_1 - \beta_1(Y_1 - p_1 - Y_2 + Kp_1)$$

as long as 1 stays ahead of 2 in money. This is decreasing in p_1, so 1 would prefer not to punish. Punishing so much that 1 is behind 2 in money is also clearly not preferred to not punishing, since such a punishment would lower 1's money and increase inequity relative to not punishing at all. Thus we can't have a NE in this sub-game where $p_1 > 0$.

- Is $p_1 = 0, p_2 > 0$ a NE of this subgame? We only need to check 2's behavior, since we are restricting ourselves away from cases where both punish. 2's payoff will be

  $$Y_2 - p_2 - \alpha_2(Y_1 - Kp_2 - Y_2 + p_2),$$

  assuming 2 does not punish so much as to get ahead of 1 in money. Note that this payoff is strictly increasing in p_2 if $\alpha_2 > \frac{1}{K-1}$. Thus 2's payoff is increasing in p_2 up until the point that monetary holdings are equal. That is,

  $$Y_1 - Kp_2 = Y_2 - p_2$$
  $$p_2 = \frac{Y_1 - Y_2}{K - 1}$$

  At this point, payoffs for both players are $Y_2 - \frac{Y_1 - Y_2}{K-1}$. Note that more punishment than this will lower payoffs for 2.

  If instead $\alpha_2 < \frac{1}{K-1}$, no punishment is optimal.

  Finally, if $\alpha_2 = \frac{1}{K-1}$, any punishment from 0 to $\frac{Y_1 - Y_2}{K-1}$ is a best response.

The case of $Y_2 > Y_1$ is symmetric.

So, in summary, the following are NE strategy profiles of the subgame starting with (Y_1, Y_2):

- If $Y_1 = Y_2$: $(0, 0)$.
- If $Y_1 > Y_2$: 1 plays 0; 2 plays 0 if $\alpha_2 < \frac{1}{K-1}$, $p_2 \in [0, \frac{Y_1 - Y_2}{K-1}]$ if $\alpha_2 = \frac{1}{K-1}$, and $p_2 = \frac{Y_1 - Y_2}{K-1}$ if $\alpha_2 > \frac{1}{K-1}$.
- If $Y_2 > Y_1$: 2 plays 0; 1 plays 0 if $\alpha_1 < \frac{1}{K-1}$, $p_1 \in [0, \frac{Y_2 - Y_1}{K-1}]$ if $\alpha_1 = \frac{1}{K-1}$, and $p_1 = \frac{Y_2 - Y_1}{K-1}$ if $\alpha_1 > \frac{1}{K-1}$.

Finally, we use these calculations to find the continuation payoffs for the initial stage of the game. We consider equilibria in the following cases:

- Suppose $\alpha_1 < \frac{1}{K-1}$ and $\alpha_2 < \frac{1}{K-1}$. Then punishment is never optimal for either player, so the SPNE are one-to-one with the NE in part (*ii*).

- Suppose $\alpha_1 \geq \frac{1}{K-1}$ and $\alpha_2 \geq \frac{1}{K-1}$. Then punishment is optimal for either player if they are behind in monetary amounts.

  Can (c, c) be part of an equilibrium in this case? If so, equilibrium payoffs are $(2a, 2a)$. If player 1 deviates to n, monetary payoffs will be $(1+a, a)$ initially, so 2 will punish such that utility is reduced to $a - \frac{1}{K-1}$ for 1, which is less than $2a$. By symmetry, 2 won't deviate for the same reason. Thus (c, c) can be part of a SPNE.

  Can (n, n) be part of an equilibrium in this case? If so, payoffs are $(1, 1)$. If 1 deviates to c, monetary amounts will be $(a, 1 + a)$ initially. 1 will then punish, such that final utility for 1 is $a - \frac{1}{K-1}$, which is clearly less than a. So 1 has no incentive to deviate. Neither does 2 by the same logic. Thus (n, n) can be part of a SPNE. This logic also rules out (n, c) and (c, n) as possible equilibrium paths.

- Suppose $\alpha_1 \geq \frac{1}{K-1}$ and $\alpha_2 < \frac{1}{K-1}$. That is, 1 wants to punish if behind, but 2 does not. Then by the same logic as the above paragraph, (n, n) can be part of a SPNE, and (n, c) and (c, n) cannot.

  What about (c, c)? If this is on the equilibrium path, payoffs are $(2a, 2a)$. If 1 deviates to n, initial monetary amounts will be $(1 + a, a)$. 2 won't punish by assumption on α_2, and neither will 1 since he is better off in terms of money than 2. This deviation will be profitable for 1 unless $2a > 1 + a - \beta_1$, or $a > 1 - \beta_1$. If 2 deviates to n, payoffs will be $(a, 1 + a)$. 1 will punish such that final payoff for 2 is $a - \frac{1}{K-1}$, so this deviation is not profitable for 2. Thus (c, c) can be part of a SPNE if $a > 1 - \beta$.

- The case of $\alpha_1 < \frac{1}{K-1}$ and $\alpha_2 \geq \frac{1}{K-1}$ is symmetric.

For each of the cases above, the first-stage actions plus the corresponding second-stage punishments constitute the set of SPNE for this game.