Economics 203: Final Review Session

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March 18, 2014

1 Logistics

- Your final exam for this class is from **3:30 to 6:30 pm on Thursday, March 20, in Room 124 of Meyer Library**.
- All problem sets have been graded and are avaible for pick-up in today's review session. I've also released solutions for all the problems, which are available on Coursework.

2 Tips and Reminders

2.1 Conceptual Tips

- Don't forget the formula for the number of strategies a player must have. As soon as you've drawn the extensive form of a game, I recommend counting info sets and actions, and then calculating how big each player's strategy set should be.
- If you are asked to give an equilibrium, make sure you list strategies for every player. An equilibrium is more than just an outcome; it is a *complete contingent plan*. It must specify an action for every player at each of that player's info sets, even if that info set is not reached on the equilibrium path.
- For any WPBE/PBE/SE, you must specify beliefs in addition to strategies. For an information set that is just a singleton, beliefs are degenerate and you don't have to specify them. But for any non-singleton information set (that is, one that contains more than one node), you need to specify beliefs for than info set as part of any WPBE/PBE/SE.

2.2 Exam Strategy

- Make it as easy as possible for me to give you credit: Write clearly, and box your answers when appropriate.
- Show your work! You will get a large part of the credit simply for setting up the problem correctly and checking the right conditions. If you just write down a wrong answer with no work, you will get the minimum possible points. You will get weakly more points if you show work.
- Work out of order if necessary, both between and within questions. If you are completely stuck on a problem or part of a problem, move on! You goal is to maximize points, so you should always be working at the part of the exam with the highest return to effort.

3 Concepts

3.1 Overview of the Course

You should be able to set up a game in normal or extensive form, with appropriate labeling of all parts of the game.

You should be able to apply the following solution concepts (and their refinements):

- (Weak) dominance
- Dominance solvability
- IDDS
- Rationalizability
- NE
- SPNE
- BNE
- WPBE
- PBE
- SE

You should be particularly familiar with the following applications, which show up often in exams:

- Price (Bertrand) competition
- Quantity (Cournot) competetion

- Repeated games
- Spence signaling model

3.2 A Brief Review of Signaling

There are several key concepts in the signaling model that you should be able to prove or recall quickly.

- Workers are paid equal to their expected value to the firm. This is because the signaling model includes a Bertrand-style wage bidding competition as the final stage. It is usually not modeled explicitly.
- Beliefs and wages are one-to-one. That is, specifying a belief for an observed activity level implies a wage, by the previous point.
- Off-path beliefs are very important: you must specify them such that the implied wages do not give either type incentive to deviate.
- In a separating equilibrium, the low type will not undertake the signaling activity. This is because is gives them no benefit (due to separation they are already known to be a low type), and is costly.
- In a separating equilibrium, the activity level of the high type is constrained by the *incentive compatibility (IC)* of the two types. There is usually (though not always) a range of such equilibria.
- In a polling equilibrium, the activity level (this time undertaken by both types) is again constrained by the IC conditions.

4 Problems

Problem 1. *Problem Bank #115 (2012 Final).*

There is an entrepreneur who would like to take her company public by selling shares of it to outside investors. The company may be of low value, θ_L, or high value, $\theta_H > \theta_L$. The entrepreneur knows the value of the company, but outside investors do not. Investors' priors are that each valuation is equally likely.

The entrepreneur, after observing the company's value, puts a fraction $q \in [0, 1]$ of her company up for sale. The market observes q (but not θ), and the shares (per unit) sell for p (an amount to be determined shortly).

The payoff for the outside investors is then $\theta q - pq$, and the entrepreneur's payoff is $pq + \frac{\theta}{2}(1 - q)$. Notice that the entire company is worth θ in the hands of outsiders ($q = 1$), but only $\frac{\theta}{2}$ in the hands of the entreprenuer ($q = 0$). This difference is what motivates the entreprenuer to sell the firm. While the source of the difference is not important for our purposes, you can think of it as reflecting either synergies with other business activities of an acquirer, or the entrepreneur's aversion to a lack of portfolio diversification.

Play proceeds as follows. First, the entreprenuer announces q. Second, multiple investors bid for the portion of the firm that is offered for sale. Third, the entreprenuer sells the offered portion of the firm to the highest bidder at the highest bid. Throughout, restrict attention to Weak Perfect Bayesian Equilibria (WPBE) in which all investors have the same beliefs conditional upon any q.

1. Suppose that upon observing q, the market forms beliefs $\mu(q)$ that the firm's value if θ_L. What will be the market price of the offered share, q?

2. Now we will focus on separating equilibria. Let $q_i = q(\theta_i)$ and $p_i = p(q(\theta_i))$ for $i = H, L$.

   (a) In any separating equilibrium, what must be the values of p_L, q_L, and p_H? Explain your answers completely.

   (b) What values of q_H could arise in a separating equlibrium?

   (c) For any given separating equilibrium, explain how to construct a belief mapping $\mu(q)$ (and an associated price mapping $p(q)$) that is consistent with the requirements of WPBE.

   (d) Explain in words why q can serve as a signal of firm value.

   (e) Which separating equilibria are ruled out by the intuitive criterion (equilibrium dominance)? Which survive? Explain your answer completely.

3. Next we consider pooling equilibria, in which entreprenuers offer the share q^* and receive price p^* irrespective of value.

   (a) What must be the value of p^*?

   (b) Is there a pooling equilibrium with $q^* = 1$? If so, what beliefs support this equilibrium?

   (c) For what other values of q^* are there pooling equilibria? Exhibit market beliefs that support those equilibria.

   (d) Which pooling equilibria are ruled out by the intuitive criterion (equilibrium dominance)? Which survive? Explain your answer completely.

Problem 2. *Problem Bank #61 (2011 Final).*

On the first day of class, the professor announces whether or not there will be a final exam. The possible announcements are Y and N, where Y stands for the announcement: "Yes, there will be an exam." N stands for the announcement: "No, there will not be an exam."

Suppose that after the announcement, the professor decides secretly (without telling the students), whether or not he will come to the final exam with a stack of exams (e) or a stack of pizzas for a party (p). Thus, the students only hear the professor's announcement (Y or N), but do NOT know whether they'll have an exam or a party (e or p). The students must decide during finals week whether or not to study (s for study, n for not study). For simplicity, assume there is one student.

(a) Draw the extensive form of this game, without any payoffs at the terminal nodes.

(b) What is each player's strategy set?

Suppose that the utilities of the professor and student depend only on whether the student studied and whether the professor wrote an exam. The announcement itself does not affect the utilities of either player. Suppose that they have the following payoffs for the given combinations of choosing whether or not to study and choosing whether or not to show up with a test:

	(e,s)	(e,n)	(p,s)	(p,n)
Professor	3	1	4	2
Student	3	1	2	4

(c) Add the payoffs to the terminal nodes of the extensive form.

(d) Does there exist a pure-strategy Nash equilibrium of this game in which the professor plays e at some information set? If so, describe such an equilibrium. If not, explain why not.

(e) Does there exist a subgame perfect Nash equilibrium of this game in which the professor plays e at some information set? If so, describe such an equilibrium. If not, explain why not.

(f) Give an example of a mixed-strategy Nash equilibrium in which the professor puts strictly positive weight on Y and N.

(g) Characterize the set of perfect Bayesian equilibria of this game.

(h) Characterize the set of sequential equilibria of this game.

Now suppose that the chair of the economics department frowns upon professors lying to students and will fire the professor if he lies; thus, the professor gets a payoff of 0 if his action does not match his announcement (regardless of whether or not the student studies). If the professor tells the truth, then his payoff is determined using the table above. The student's payoff is determined by the table above regardless of whether the professor has told the truth or lied.

(i) Draw the extensive form again, but modify the payoffs at the terminal nodes where appropriate. Note, the shape of the extensive form diagram hasn't changed.

(j) Now, using the new diagram, characterize the set of perfect Bayesian equilibria.

Problem 3. *Problem Bank #99 (2013 Final).*

In the market for new economists, there are two departments $i \in \{1, 2\}$ and three recent graduates $j \in \{1, 2, 3\}$. When recent grad j goes to department i, the department increases their reputational payoff by $i \cdot j$. Thus, department i's payoff for being matched with graduate j, when offering wage p is $i \cdot j - p$. Assume throughout that firms have to offer $p \geq 0$.

To save costs, the economics profession institutes a centralized matching system to pair recent Ph.D. graduates with their future employers. Instead of offering personalized wages, each department submits a single wage and whichever department submits the highest wage gets the best new graduate (here $j = 3$); the other firm gets the next best graduate (here $j = 2$). If they offer the same price, department 2 gets the higher quality graduate.

1. What are the strategy sets of the two departments if Department 1 has to submit their price *before* Department 2? What if the two departments submit their wage simultaneously?

   For the rest of the problem, assume that that wages are submitted simultaneously.

2. Is there a pure strategy Nash equilibrium? If so, characterize it. If not, explain why not.

3. Now we will look for a mixed strategy Nash equilibrium. Let F_i denote the CDF governing department i's choice of wage. You may assume that the support of F_i is an interval $[0, p']$ for some $p' > 0$ (where p' is the same for both departments), that F_1 has an atom at 0 but nowhere else, and that F_2 is atomless.

   (a) Write an expression for the expected payoff of department 2 as a function of the wage it sets, given F_1. Also write an expression for the expected payoff of department 1 as a function of the wage it sets, given F_2.

   (b) What is Department 1's equilibrium profit? Using your answer, solve for p' and department 2's equilibrium CDF, $F_2(p)$.

   (c) What is Department 2's equilibrium profit? Using your answer, solve for department 1's equilibrium CDF, $F_1(p)$?

   (d) Explain why, in any mixed strategy equilibrium, neither department uses a CDF with an atom at any wage greater than zero.

4. Suppose that the departments repeat the game an infinite number of times (with new students each time). If the departments use the average payoff criterion, what is the set of possible payoffs for the two departments?

   Now suppose that graduate three has an outside offer of $\overline{p}$. Thus, if the highest wage offered is less than $\overline{p}$, then the department that offers the highest wage is matched with Graduate 2 (and thus receives a reputational benefit of $2 \cdot i$) and the department that offers the lowest wage is matched with Graduate 1. On the other hand, if the highest wage offered is greater than or equal to $\overline{p}$, the matching process is the same as above: the department with the highest wage offer gets Graduate 3 and the other department gets Graduate 2.

5. Suppose $\overline{p} \in [1, 2]$. Is there a pure strategy Nash equilibrium? If so, identify it; if not, explain why not.

6. Now suppose that $\overline{p} \in (0, 1)$. Solve for a mixed strategy equilibrium. You may assume that both departments choose prices in $(0, \overline{p})$ with zero probability, that department 1 (and only department 1) chooses $p = 0$ with positive probability, that department 2 (and only department 2) chooses $\overline{p}$ with strictly positive probability, and that the departments otherwise randomize over an interval $(\overline{p}, p']$. (Hint: follow the same steps as in part 3, but also check to make sure that neither department has an incentive to deviate to any $p \in (0, \overline{p})$.)

5 Solutions

Solution 1.

1. For the usual reasons, Bertrand competition among bidders assures that $p = [\mu(q)\theta_L + (1 - \mu(q))\theta_H]$.

2. (a) *In a separating equilibrium $\mu(q_H) = 0$ and $\mu(q_L) = 1$, so $p_L = \theta_L$ and $p_H = \theta_H$. Furthermore, we must have $q_L = 1$. If we had $q_L < 1$, the payoff to the entreprenuer in charge of a low-value firm would be $q_L\theta_L + (1 - q_L)\frac{\theta_L}{2} < \theta_L$. If the entreprenuer deviates to $q = 1$, the inference can be no worse that that his firm is of low value, so his payoff can be no lower than, θ_L, which means he is strictly better off.*

   (b) *In a separating equilibrium, the entreprenuer in charge of a low-value firm receives a payoff of θ_L. If he instead sells the fraction q_H of his firm, he receives a payoff of $\theta_H q_H + (1 - q_L)\frac{\theta_L}{2}$. Therefore, we must have*

$$\theta_H q_H + (1 - q_H)\frac{\theta_L}{2} \leq \theta_L,$$

or

$$q_H \leq \frac{\theta_L}{2\theta_H - \theta_L} \equiv q_1.$$

Notice that $q_1 \in (0,1)$. Likewise, in a separating equilibrium, the entreprenuer in charge of a high-value firm receives a payoff of $\theta_H q_H + (1-q_H)\frac{\theta_H}{2}$. If instead he sells all of his firm, he receives a payoff of θ_L. Therefore, we must have

$$\theta_H q_H + (1-q_H)\frac{\theta_H}{2} \geq \theta_L,$$

or

$$q_H \geq \frac{2\theta_L - \theta_H}{\theta_H} \equiv q_2.$$

Notice that $q_1 > q_2$ because $q_1 - q_2 = \frac{\theta_H - \theta_L}{\theta_H} > 0$. Any $q_H \in [\max\{0, q_2\}, q_1]$ can arise in a separating equilibrium.

(c) *The easiest way to do this is simply to set $\mu(q) = 1$, and thus $p(q) = \theta_L$, for all $q \neq q_H$. All the non-equilibrium choices are then less attractive than q_L.*

(d) *In this model, all entreprenuers want to sell all of their firms. Retaining a portion of the ownership is costly, but it is less costly if the firm is of high quality. Therefore, an entreprenuer of a high quality firm can signal quality by showing a willingness to retain a larger share.*

(e) *Only the efficient equilibrium, with $q_H = q_1$, survives. For any smaller q_H, there is by construction no inference $\mu \in [0,1]$ that would induce an entreprenuer with a low-value firm to choose $q \in (q_H, q_1)$ over his equilibrium choice, but the inference $\mu = 1$ would induce an entreprenuer with a high-value firm to choose the same q over her equilibrium choice. Thus, the equilirium does not satisfy the intuitive criterion. In the case of $q_H = q_1$, there is no inference that would induce a θ_H entreprenuer to choose $q < q_1$, and there are inferences that would induce a θ_L entreprenuer to choose $q > q_1$, so the intuitive criterion has no bite, and the equilibrium survives.*

3. Next we consider pooling equilibria, in which entreprenuers offer the share q^* and receive price p^* irrespective of value.

   (a) *Based on part (1), we must have $p^* = \frac{\theta_H + \theta_L}{2}$.*

   (b) *Yes; it can be supported, for example, by the belief $\mu(q) = \frac{1}{2}$ for all q.*

   (c) *The beliefs most favorable to supporting a pooling equilibrium are $\mu(q) = 1$ for all $q \neq q^*$. In that case, the best deviation is to $q = 1$, which is not attractive for a θ_k entreprenuer as long as*

$$p^* q^* + (1-q^*)\frac{\theta_k}{2} \geq \theta_L.$$

Clearly, if this holds for $k = L$, it also holds for $k = H$. For $k = L$, the constraint becomes

$$\begin{aligned} q^* &\geq \frac{\theta_L}{2p^* - \theta_L} \\ &= \frac{\theta_L}{\theta_H}. \end{aligned}$$

(d) *None of them survive. A θ_L entreprenuer is unwilling to choose q rather than q^* for any inference if*

$$p^* q^* + (1 - q^*)\frac{\theta_L}{2} \geq \theta_H q + (1 - q)\frac{\theta_L}{2}$$

or

$$q \leq q^* \left[\frac{2p^* - \theta_L}{2\theta_H - \theta_L}\right] \equiv Q(q^*).$$

Notice that $Q(q^*) \in (0, q^*)$. For q slightly less than $Q(q^*)$, a θ_L entreprenuer would not choose q over his equilibrium outcome for any feasible inference, while a θ_H entreprenuer would choose over his equilibrium outcome for the inference θ_L (this follows from single-crossing and the near-indifference for entreprenuers with low-value firms, but it can also be checked directly). Hence, the inference from such q must be $\mu(q) = 0$, which implies $p(q) = \theta_H$. But then a θ_H entreprenuer would indeed deviate to q.

Solution 2.

(a) See Figure 1.

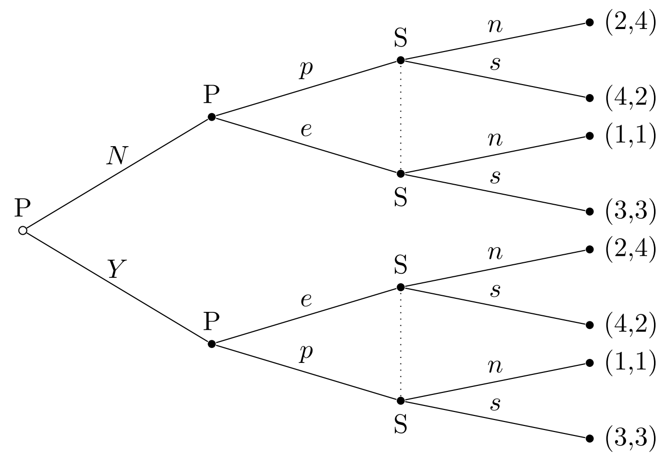

Figure 1: The extensive form for the game in 2(a).

(b) The professor has 3 info sets: the root node, after he says Y, and after he says N. At each info set he chooses from among two actions, and so he will have 8 strategies. His strategy set is $S_P = \{Y, N\} \times \{e, p\} \times \{e, p\}$.

The student has just two information sets: After the professor says Y and after the professors says N. At each info set she has two possible actions, for a total of four strategies. We have $S_S = \{s, n\} \times \{s, n\}$.

(c) See Figure 1.

(d) I think the easiest and most reliable way to answer this question is to write down the normal form. It is large, but relatively fast to fill out since there is must repetition. From the normal form, you can immediately see that we have the following pure-strategy NE: $(Ype, nn), (Ypp, nn), (Nep, nn)$, and (Npp, nn).

(e) Note that the game has two proper subgames: after Y and after N. These two subgames are identical, and the unique NE of each is (p, n). Thus the professor cannot play e at any point in a SPNE.

(f) From our normal form in (d), we can see that there are several possible ways to do this. One such MSNE: the professor strictly mixes between Ypp and Npp, and the student plays nn. Note that the student has no incentive to deviate, as she is getting her best payoff, 4. The professor also will not deviate, as he cannot do better than a payoff of 2 if the student is playing nn. Note that this is also a SPNE.

(g) First, note that we only need to consider strategies that are SPNE. The only SPNE are where the professor mixes between Ypp and Npp and the student plays nn with certainty. Thus, it remains to find beliefs that make our SPNE sequentially rational and that follow Bayes' rule.

First, some notation. Let μ_Y be the student's belief that the professor has decided to play e after announcing Y. Similarly, let μ_N be her belief that the professor has decided to play e after announcing N.

Note that the professor is clearly sequentially rational. Given the student's actions, the professors is indifferent between Ypp and Npp. For the student to be sequentially rational, she must be best-responding to her beliefs. So, for nn to be best response, we need $1\mu_Y + 4(1-\mu_Y) \geq 3\mu_Y + 2(1-\mu_Y)$, or $\mu_Y \leq \frac{1}{2}$, as well as $\mu_N \leq \frac{1}{2}$.

Next, consider Bayes' rule. If Y is played with positive probability, then the student's info set after Y is on the equilibrium path, and we can thus calculate that $\mu_Y = 0$. Similarly, if N is played with some probability, we must have $\mu_N = 0$.

These conditions on beliefs support $(\alpha Ypp + (1-\alpha)Npp, nn)$ as a *weak* perfect Bayesian equilibrium for any $\alpha \in [0,1]$. For a PBE, we also need a WPBE in every subgame. In this case there are two, as we noted earlier, and they are identical. In the subgames, the specified strategies are (p, n), so beliefs must be $\mu_Y = \mu_N = 0$ from Bayes' rule. (Note that for the strategies restricted to these subgames, the student's information set is always reached.)

So in summary, we have the following set of PBE:

$$\left\{ (\alpha Ypp + (1-\alpha)Npp, nn), \mu_Y = \mu_N = 0 | \alpha \in [0,1] \right\}.$$

(h) We know that the set of SE will be contained in the set of PBE. We need beliefs to be consistent, so consider a sequence δ^n of strictly mixed strategies. In particular, let δ_Y^n be the probability that the professor plays Y, and similar for δ_{eY}^n and δ_{pY}^n. Note also that we must have $\delta_{eY}^n \to 0$ and $\delta_{eN}^n \to 0$. From Bayes' rule we get

$$\mu_Y^n = \frac{\delta_Y^n \delta_{eY}^n}{\delta_Y^n \delta_{eY}^n + \delta_Y^n (1 - \delta_{eY}^n)} = \delta_{eY}^n \to 0.$$

Thus the only possible consistent beliefs are $\mu_Y = 0$. A similar procedure shows that we must have $\mu_N = 0$ as well.

Thus we have the following set of SE: $\left\{ (\alpha Ypp + (1-\alpha)Npp, nn), \mu_Y = \mu_N = 0 | \alpha \in [0,1] \right\}$. This is the same as the set of PBE above.

(i) See Figure 2.

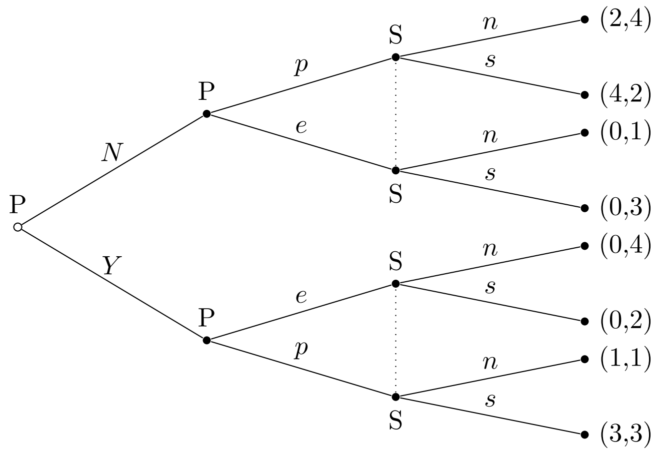

Figure 2: The extensive form for the game in 2(i).

(j) Note that in the Y subgame, the unique NE is (e, s), while in N subgame, the unique NE is (p, s). Given these continuation equilibria, the professor would rather choose Y at the beginning. Thus the unique SPNE is (Yep, sn). To find the possible PBE, we simply need to see which beliefs are allowed for this strategy profile.

First, Bayes' rule gives us that $\mu_Y = 1$. Next, note that the professor is sequentially rational, since Y then e gives him the highest payoffs given that the student is playing sn. The student is sequentially rational for playing s after Y given her beliefs that $\mu_Y = 1$. After N, we need her beliefs to be such that p is a best response: $3\mu_N + 2(1 - \mu_N) \leq 1\mu_N + 4(1 - \mu_N)$, or $\mu_N \leq \frac{1}{2}$.

However, as before we need to further check that the specific beliefs and strategies imply a WPBE in each subgame. This immediately requires $\mu_N = 0$ by Bayes' rule. Thus we have a unique PBE: $((Yep, sn), (\mu_Y = 1, \mu_N = 0))$.

Solution 3.

1. If Department 1 submits before Department 2, Department 1's strategy set is $\{p | p \geq 0\}$ and Department 2's strategy set is $\{f | f : [0, \infty) \to [0, \infty)\}$, i.e. the set of functions that take positive values and output positive values. If they both submit their wage offers simultaneously, both strategy sets are $\{p | p \geq 0\}$.

2. No. First, suppose there was a pure strategy Nash equilibrium where $p_i < p_j$. Then Department j has an incentive to bid $p_j - \frac{p_j - p_i}{2}$. Next, suppose there was a pure strategy Nash equilibrium with $p_i = p_j$. Then firm i has an incentive to play $p_i + \epsilon$ for some small ϵ. Thus, there is no

pure strategy Nash equilibrium.

3. (a) For $i \in \{1,2\}$, their profit is $\pi_i(p) = i \cdot \Big[3F_{-i}(p) + 2\big(1 - F_{-i}(p)\big)\Big] - p$. Thus, $F_{-i}(p) = \frac{\pi_i + p}{i} - 2$

   (b) We can assume that Department 2 has no atom at 0. So $\pi_1(0) = 1 \cdot 2 - 0 = 2$. We also get to assume that Department 1 has an atom at zero, so by the mixed strategy indifference condition, $\pi_1 = 2$. Thus, Department 2's CDF is $F_2(p) = p$. This also shows that $p' = 1$.

   (c) We know that Department 2 is guaranteed at getting worker 3 when playing p'. Given that $p' = 1$, $\pi_2 = 2 \cdot 3 - 1 = 5$. Thus, $F_1(p) = \frac{5+p}{2} - 2 = \frac{1}{2} + \frac{p}{2}$.

   (d) It cannot be the case that both have an atom at a $\hat{p} > 0$ for the same reasons as why there cannot be a PSNE. Suppose that Department i, and only Department i, has an atom at $\hat{p} > 0$. Department j's utility function is then discontinuous at $\hat{p}$, meaning that they will not play in some $\epsilon > 0$ range below $\hat{p}$. Given that Department j doesn't play in $(\hat{p} - \epsilon, \hat{p})$ and doesn't have an atom at $\hat{p}$, Department j's utility of playing $\hat{p} - \epsilon$ is strictly greater than playing $\hat{p}$. Thus, Department j cannot have an atom at $\hat{p}$.

4. The Folk theorem says that we can get any payoff in the convex hull of the stage game payoffs that gives each player a weekly higher payoff than their minmax payoff. It's easiest to draw this answer to find the convex hull of the stage game. Each Department can get worker 2 by playing zero, so the minmax payoffs are $(\pi_1, \pi_2) = (2, 4)$.

5. Yes. If $\bar{p} \in [1, 2]$, Department 2 playing $\bar{p}$ and Department 1 playing 0 is a pure strategy Nash equilibrium. You can easily check that neither firm has an incentive to deviate.

6. If they both play $p \geq \bar{p}$, the payoffs (and thus implied $F_i(p)$) are identical to part 3. The only possible difference in the upper limit p' and the payoffs. Department 1 still has an atom at 0 and is guaranteed worker two when playing that; thus $\pi_1 = 2$ as before. This implies that in $(\bar{p}, p')$, $F_2(p) = p$. We then get that $p' = 1$, as before, but that now Department 2 has an atom at $\bar{p}$ equal to $\bar{p}$. Likewise, since p' is the same as in part 3, π_2 is the same and thus $F_1(p)$ is the same in the range $[\bar{p}, 1)$. Their atom at 0 is now of size $\frac{1}{2} + \frac{\bar{p}}{2}$. Finally, we need to check that neither firm has an incentive to play something outside of the range. They certainly won't play something greater than p', because they are already guaranteed the best worker when playing p'. So we need only check if they have an incentive to deviate in $(0, \bar{p})$. If anyone would have an incentive to deviate, it would be Department 2, and their best payoff from playing in $(0, \bar{p})$ is to play ϵ. That guarantees them worker 2 (either Department 1 plays lower, or Department 1 gets the worker 3 by playing higher than $\bar{p}$) so

their payoff is 4. As we showed, in equilibrium $\pi_2 = 5$, so they have no incentive to deviate.