Economics 203: Section 1

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1 Logistics

1.1 Contact Information

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Office hours: Thursday 1:30 - 3:30, Landau 245

1.2 Section Meetings

Time: Tuesdays, 9:00 am to 10:50 am
Location: Landau 218

Sections will begin with a quick review or discussion of key topics from the previous week. I will usually have a few comments prepared, but this is also a great time to ask me questions. If possible, try to email me your questions by the day before section, so that I have time to prepare a thorough answer. We will then dedicate the rest of the section to solving problems.

1.3 Resources

- Doug's lecture notes, posted on Coursework[1]
- My section notes, posted on Coursework and my personal page
- The problem bank, posted on Coursework
- Problem bank solutions, posted on Coursework after problem sets turned in
- In addition to the textbooks in Doug's syllabus, I recommend Osbourne and Rubinstein's *A Course in Game Theory*.

1.4 Problem Sets

Problem sets will be assigned by me on Fridays, to be turned in the following Friday at 5:00 pm in my box on the second floor of the Landau building. Problem sets will consist of a subset of the problem bank problems. I will try to choose the problems so that the requisite material will be covered in the previous week's lectures, though sometimes the material will spill into the lecture on the Tuesday of the week the problems are due.

[1] See `https://coursework.stanford.edu/portal/site/W12-ECON-203-01`

Problem sets will be graded as described by Doug: I will assign you a check if you write down a serious attempt at every part of every question assigned. It is totally OK if the answer is wrong, as long as it looks like you attempted the given problem. It is also OK if you copy word-for-word from a previous year's solutions. If you get a check for every assignment this quarter, you will get a half-grade bump in your course grade (e.g. from an A- to an A).

1.5 Other Information

- Doug means what he says about his email habits. Don't be shy about emailing him multiple times if you don't get a response within a day.
- Please help out my email load by putting "203" somewhere in your subject line when sending me a message.
- The date for the final exam will be scheduled by the university sometime in the next few weeks.

2 Review

2.1 Course Outline

The sections of the course are organized around two key concepts: complete vs incomplete games, and static vs dynamic games:

1. Strategic environments: describe how we model social interactions
2. Static + complete information
3. Static + incomplete information
4. Dynamic games overview
5. Dynamic + complete information
6. Repeated games (a special case of dynamic + complete info)
7. Dynamic + incomplete information (e.g., signaling games)

2.2 Why Solution Concepts?

Broadly speaking, once we have written down a game in either extensive or normal form, we can see all of the possible outcomes of the games. There may be a large or even infinite number of outcomes, so our job as game theorists is to predict which outcomes are likely to happen. Hopefully we can get a unique prediction, but at the very least we hope to identify a very small subset of all the outcomes. This is what we mean by a **solution**. Thus, a **solution concept** is a rule that, given the game, predicts which outcome or outcomes will actually come to pass.

2.2.1 Solution Concept: Dominant Strategy Solution

Definition 1. A strategy s_i is a **dominant strategy** for player i iff for all $\hat{s}_i \in S_i$ such that $\hat{s}_i \neq s_i$ and for all $s_{-i} \in S_{-i}$, we have $g_i(s_i, s_{-i}) > g_i(\hat{s}_i, s_{-i})$.

Note. In plain English: A dominant strategy is one that is the best response for a player for every possible strategy their opponent is playing.

Definition 2. If all players have a dominant strategy, we say that the game has **dominant strategy solution**.

	NF	F
NF	$(-2,-2)$	$(-5,-1)$
F	$(-1,-5)$	$(-4,-4)$

Figure 1: Prisoner's Dilemma

Example 1. The game Prisoner's Dilemma (Figure 1) has a dominant strategy solution. To see why, note that if your opponent is playing *Fink*, your best response is *Fink* (since $-4 > -5$). If your opponent is playing *Not Fink* your best response is still *Fink* (since $-1 > -5$). It is easy to see this graphically by plotting the payoff to each strategy as a function of the other players strategy, as in Figure 2.

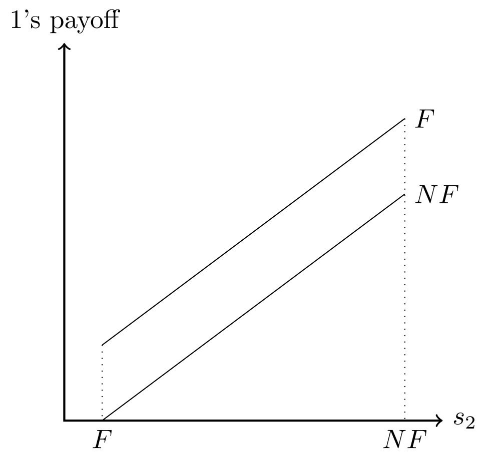

Figure 2: Seeing a dominant strategy graphically.

2.2.2 Solution Concept: Dominance Solvable

Define $K_i = |S_i|$ for each player i. Label the K_i strategies for i by $k = 1, 2, \ldots, K_i$. A probability measure ρ on the strategy set S_i is a vector $\rho = (\rho_1, \ldots, \rho_{K_i})$ such that $\rho_m \geq 0$ for all $1 \leq m \leq K_i$ and $\sum_{m=1}^{K_i} \rho_m = 1$.

Definition 3. A strategy s_i^k is a **dominated strategy** for player i iff there exists a measure ρ on S_i such that

$$\sum_{m=1}^{K_i} \rho_m g_i(s_i^m, s_{-i}) > g_i(s_i^k, s_{-i})$$

for all $s_{-i} \in S_{-i}$.

Note. In plain English: A strategy is dominated if the player can do strictly better with some (possibly degenerate) mixture of strategies, for every possible strategy their opponent could be playing.

Definition 4. If iterative deletion of dominated strategies yields a unique outcome, we say that the game is **dominance solvable**.

Note. In IDDS, we delete only one player's strategy (or strategies) at a time, even if at a given round both have dominated strategies.

Example 2. Consider the normal form game from Doug's notes, reproduced in Figure 3. It is clear that both players do not have a dominant strategy. Note, however, that 2's strategy c is strictly dominated by

	a	b	c
A	$(4, 10)$	$(3, 0)$	$(1, 3)$
B	$(0, 0)$	$(2, 10)$	$(10, 3)$

Figure 3: A normal form game that is dominance solvable.

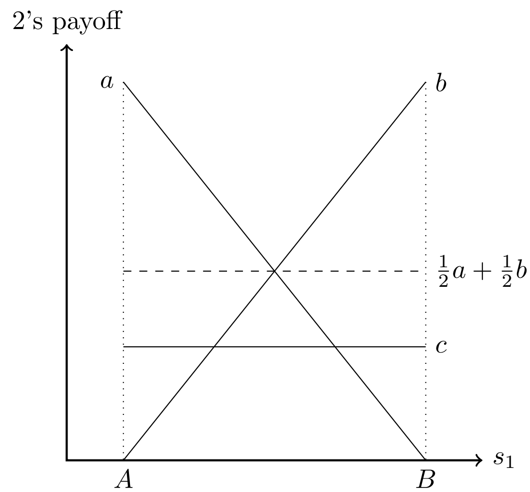

Figure 4: Seeing a dominated strategy graphically.

playing and even mixture of a and b. This is easily seen graphically in Figure 4. Deleting this strategy begins a chain of IDDS that eventually leads to the outcome (A, a).

This example highlights the fact that the payoffs listed are the von Neumann-Morgenstern (vNM) utilities for the given outcome. This means that the risk aversion for each player, if any, is already built into the payoff numbers, and so players rank outcomes or mixtures of outcome based solution on their expected value.

3 Problems

3.1 Split-The-Pie

Consider the following situation. Two people (A and B), want to split a delicious pie. A cuts the pie into two pieces, and then B picks one of those two pieces to consume. A gets to consume the other piece. For this problem, let's assume that A can only cut the pie in half, or not cut it at all. Let's also assume that each player's vNM utility is equal to the fraction of the pie they consume.

1. Draw the extensive form for this game. (You can assume the pie is of size 1.)
2. What are the strategy sets for the two players?
3. Draw the normal form of this game.
4. Are there any dominated strategies? Is the game dominance solvable?

3.2 Split-The-Pie Redux

Consider the situation in 3.1 above, but now A can cut the pie into two pieces any way she wants.

1. Draw the extensive form for this game.

2. What are the strategy sets for the two players?

3. Are there any dominated strategies? Is the game dominance solvable?

3.3 Guessing Game

There is a class of simultaneous-move games known as *two-person guessing games.* In these games, each player's move is a guess $x_i \in [a_i, b_i]$. Each player has a *target*, $t_i = \alpha_i x_j$, where α_i is called i's *multiplier.* Payoff for player i is $-|x_i - t_i|$. That is, each player wants guess as close as possible to their multiplier times the other player's guess.

For this problem, assume $a_1 = a_2 = 0$, $b_1 = b_2 = 100$, and $\alpha_1 = \alpha_2 = \frac{1}{4}$.

1. Do the players have any dominated strategies?

2. Is the game dominance-solvable?

4 Solutions

4.1 Split-The-Pie

1. We have two players, A and B. Let's say that the pie is cut into two pieces, Y and Z. Then A's choice is what size to make piece Y, after which B's choice is whether to pick piece Y or piece Z. Thus the extensive form of the game can be drawn as in Figure 5. Note that the extensive form tells us the game tree, any information sets, and the payoffs. The set of players is implicit in both the tree and the vector listing the payoffs.

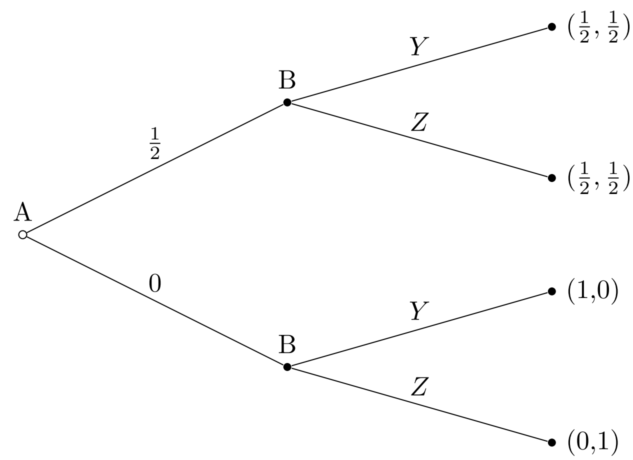

Figure 5: The extensive form of Split-The-Pie.

2. Player A has one information set, so in this case actions and strategies will coincide. Thus $S_A = \{0, \frac{1}{2}\}$. Player B has two information sets: one after A plays $\frac{1}{2}$ and one after A plays 0. Thus each member of S_B will be an ordered list: the first term will be B's action after the first information set, and the second term will be the action to follow the second information set. Thus $S_B = \{(Y,Y),(Y,Z),(Z,Y),(Z,Z)\} = \{Y,Z\} \times \{Y,Z\}$.

3. The normal form is given in Figure 6. Player A is the row player and player B is the column player. Note that the normal form tells us all possible strategies for all players, as well as the payoffs. As in

	(Y,Y)	(Y,Z)	(Z,Y)	(Z,Z)
$\frac{1}{2}$	$(\frac{1}{2},\frac{1}{2})$	$(\frac{1}{2},\frac{1}{2})$	$(\frac{1}{2},\frac{1}{2})$	$(\frac{1}{2},\frac{1}{2})$
0	$(1,0)$	$(0,1)$	$(1,0)$	$(0,1)$

Figure 6: The normal form of Split-The-Pie.

the extensive form, the set of players is implicit in the payoffs (and in the fact that we can write the game as a matrix).

4. A does not have a dominated strategy. To see why, note that 0 is not dominated because it is the best response when B plays (Y,Y), for example. Similarly, $\frac{1}{2}$ is not dominated because it is the best response when B plays (Z,Z), for example

   B does not have a dominated strategy either, since all strategies give the same payoff when A plays $\frac{1}{2}$.

   Thus, there are no dominated strategies and the game is not dominance solvable.

4.2 Split-The-Pie Redux

1. Let's say that A chooses x, the size of piece Y. Then the extensive form can be drawn as in Figure 7.

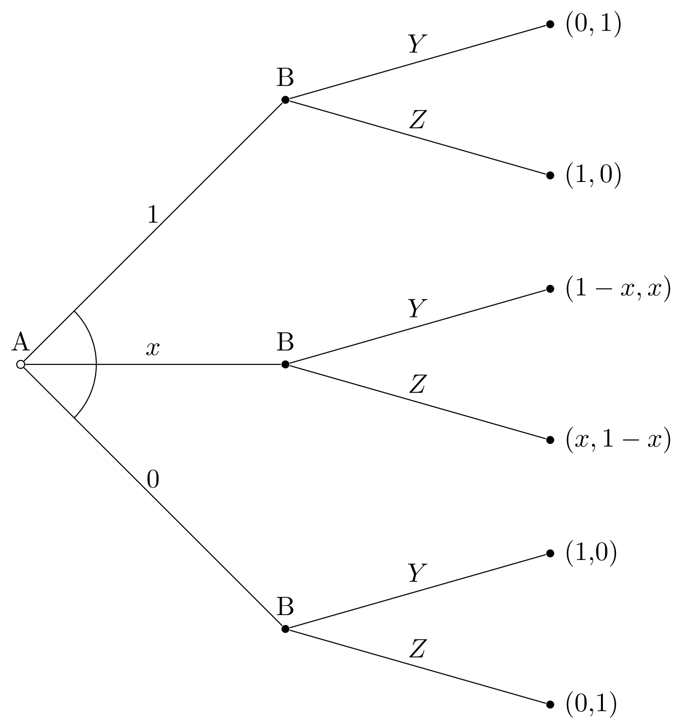

Figure 7: The extensive form of Split-The-Pie Redux.

2. A can choose to cut the pie in any size, so $S_A = [0,1]$. B's strategy is a little more complicated: for any split that A gives him, he must say whether he would like piece Y or piece Z. Thus, a strategy for B is a function from the inteval $[0,1]$ to the set $\{Y,Z\}$. That is, S_B is the set of all functions f such that $f : [0,1] \to \{Y,Z\}$.

3. There are no dominated strategies for A. To see why, suppose the some strategy $\bar{x} \in [0,1]$ is dominated. Then there exists a measure ρ on $[0,1]$ such that

$$\int \rho(x) g_A(x,Y)dx > g_A(\bar{x},Y) \text{ and} \tag{1}$$

$$\int \rho(x) g_A(x,Z)dx > g_A(\bar{x},Z). \tag{2}$$

But since $g_A(x,Y) = (1-x)$ and $g_A(x,Z) = x$, we have the following implications:

$$\begin{aligned} \int \rho(x) g_A(x,Y)dx > g_A(\bar{x},Y) &\Rightarrow \int \rho(x)(1-x)dx > (1-\bar{x}) \\ &\Rightarrow 1 - \int \rho(x) x dx > (1-\bar{x}) \\ &\Rightarrow \int \rho(x) x dx < \bar{x} \\ &\Rightarrow \int \rho(x) g_A(x,Z)dx < g_A(\bar{x},Z), \end{aligned}$$

which is a contradiction. This $\bar{x}$ must not be dominated.

There are also no dominated strategies for B. If A plays $x = \frac{1}{2}$, then B is indifferent between all strategies.

Thus, there are no dominated strategies, and the game is not dominance solvable.

4.3 Guessing Game

1. Playing $x_i > 25$ is dominated by playing 25 for $i \in \{1,2\}$. To see why, note the following:

$$x_i > 25 \tag{3}$$

$$\Rightarrow x_i - \frac{1}{4}x_j > 25 - \frac{1}{4}x_j \quad \forall x_j \tag{4}$$

$$\Rightarrow \left| x_i - \frac{1}{4}x_j \right| > \left| 25 - \frac{1}{4}x_j \right| \quad \forall x_j \qquad \text{since } 25 - \frac{1}{4}x_j \geq 0 \tag{5}$$

$$\Rightarrow -\left| 25 - \frac{1}{4}x_j \right| > -\left| x_i - \frac{1}{4}x_j \right| \quad \forall x_j \tag{6}$$

This last line is the definition of $x_i > 25$ being a dominated strategy!

2. In the first round, 1 eliminates strategies above $25 = \left(\frac{1}{4}\right) 100$. In the second round, 2 can then eliminate strategies greater than $\left(\frac{1}{4}\right)^2 100$. In the third round, 1 eliminates strategies greater than $\left(\frac{1}{4}\right)^3 100$, and so on. After an infinite number of rounds, all positive strategies are eliminated for both players. Thus the dominance solvability outcome is $x_1 = x_2 = 0$.